Use of continuation method for nonlinear systems depending on parameters and their bifurcations

Igor Schreiber, Prague Institute of Chemical Technology

Parameter sensitivity of dynamical systems described by differential (or difference) equations can be studied by the continuation method. Given a certain type of solution of the dynamical equations for a specific parameter value, continuation provides a smooth one-parameter family of solutions. The two main types of solutions that may be continued are: (i) time-independent solutions or steady states, and (ii) time dependent solutions which must be further specified by boundary conditions. The simplest example of the latter type are periodic solutions. While continuation of steady states is rather straightforward, boundary value problems imply additional difficulty of determining the solution within the time domain by using either discretization or shooting method. The branch of solutions obtained by continuation may contain singular codimension-one points, such as turning points or points where the solution becomes unstable. Such bifurcation points may be localized by including relevant bifucation conditions into the set of equations which specify the continuation problem. By continuing with respect to a new parameter, we obtain a curve of bifurcation points which may contain singular points of codimension two. By performing simultaneous stability and bifurcation analysis during continuation we can ultimately construct bifurcation diagrams in one, two or even more parameters. These diagrams are useful by providing a detailed insight into various kinds of dynamics of the studied systems. An overview of numerical methods used to obtain a bifurcation diagram will be provided.